SAMPLE SYLLABUS #1

AP® Calculus BC

Curricular Requirements

CR1	The students and teacher have access to a college-level calculus textbook, in print or electronic format.	*See page:* 4
CR2	The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.	*See page:* 2
CR3	The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes.	*See pages:* 16, 17
CR4	The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations.	*See page:* 16
CR5	The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification.	*See page:* 16
CR6	The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation.	*See page:* 16
CR7	Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.	*See pages:* 3, 16
CR8	The course provides opportunities for students to use calculus to solve real world problems.	*See page:* 17

Advanced Placement
Calculus BC Sample Syllabus #1

Overview

AP® Calculus BC satisfies all the requirements designed by the College Board and is equivalent to two semesters of college level calculus. This course syllabus is aligned to the *AP Calculus AB and BC Course and Exam Description* (CED) released by the College Board in 2019. Students enrolled in this course have completed precalculus and have chosen to take BC Calculus (in lieu of AB Calculus, which our school also offers). Students are required to take AP Calculus BC Exam in May. If students cannot afford to pay for the exam, the school will pay for the exam.

The course is designed around the three "Big Ideas" of calculus, including:

Big Idea #1: Change

Big Idea #2: Limits

Big Idea #3: Analysis of Functions

The College Board's CED is broken down into 10 units, and my course follows the sequencing/pacing of these 10 units. The three big ideas of calculus are included in the units as reflected in the CED. **CR2**

UNIT 1: Limits and Continuity (~3 weeks)

UNIT 2: Differentiation: Definition and Fundamental Properties (2–3 weeks)

UNIT 3: Differentiation: Composite, Implicit, and Inverse Functions (2–3 weeks)

UNIT 4: Contextual Applications of Differentiation (~2 weeks)

UNIT 5: Analytical Applications of Differentiation (2–3 weeks)

UNIT 6: Integration and Accumulation of Change (~4 weeks)

UNIT 7: Differential Equations (2–3 weeks)

UNIT 8: Applications of Integration (3–4 weeks)

UNIT 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions (~3 weeks)

UNIT 10: Infinite Sequences and Series (4–5 weeks)

CR2
The syllabus must include an outline of course content by unit title or topic using any organizational approach with the associated big idea(s) to demonstrate the inclusion of required course content. All three big ideas must be included: Change, Limits, and Analysis of Functions.

Student Practice

Throughout each unit, **Topic Questions** will be provided to help students check their understanding. The Topic Questions are especially useful for confirming understanding of difficult or foundational topics before moving on to new content or skills that build upon prior topics. Topic Questions can be assigned before, during, or after a lesson, and as in-class work or homework. Students will get rationales for each Topic Question that will help them understand why an answer is correct or incorrect, and their results will reveal misunderstandings to help them target the content and skills needed for additional practice.

At the end of each unit or at key points within a unit, **Personal Progress Checks** will be provided in class or as homework assignments in AP Classroom. Students will get a personal report with feedback on every topic, skill, and question that they can use to chart

their progress, and their results will come with rationales that explain every question's answer. One to two class periods are set aside to re-teach skills based on the results of the Personal Progress Checks.

An extra lab period each week is devoted to an appropriate calculator activity, multistep word problems, Topic Questions, Personal Progress Checks, and/or free-response questions (FRQ's) from released AP Calculus BC Exams. Emphasis is placed on problem solving, using the calculus in new settings, and helping students to see the connections among the big ideas and the major themes in calculus. FRQs, which emphasize real-world applications of the calculus, are selected for discussion during this lab period.

The course is also designed around the four Mathematical Practices in AP Calculus outlined in the 2019 CED including:

Practice #1: Implementing Mathematical Processes

Practice #2: Connecting Representations

Practice #3: Justification

Practice #4: Communication and Notation

Course Objectives

At the end of the course, students should be able to solve a variety of real-world problems using limits, derivatives, integrals, and series. Students are shown the interrelationships of these four major themes/threads throughout the course. The course teaches the students how to communicate their mathematical reasoning using proper mathematical terminology in complete sentences. Students are instructed how to answer problems in the context of the problem, both verbally and in written sentences/paragraphs, using appropriate measurement units.

Prerequisites

All students who are taking AP Calculus BC have completed precalculus and have a firm understanding of:

- Functions – their graphs and behaviors
- Trigonometry
- Logs and Natural Logs
- Transformations and Translations
- The use of their graphing calculator to solve problems
- The value of the Rule of Four to solve problems (analytical/algebraic, numerical, graphical, verbal/communication)
- Transcendental Functions

These and other prerequisite topics/skills are briefly reviewed, as needed, during the year to help students make valuable connections between the big ideas.

Technology

- All students are expected to have a TI-83, 83+, 84, or 84+ for their use in class and for homework assignments. For students that cannot afford a calculator, our school will loan a calculator to that student for the course. **CR7**
- All students have access to the computer labs at our school.
- The graphing calculator is used every day in class and students are instructed daily on how to use this technology to help them understand the various calculus concepts and to connect concepts and different representations.

CR7

The syllabus includes a statement that each student has individual access to an approved graphing calculator.

AND

The syllabus must include a description of at least one activity in which students use graphing calculators to:

- graph functions
- solve equations
- perform numerical differentiation
- perform numerical integration
- explore or interpret calculus concepts

- Students are exposed to numerous calculus applets during the course, and I have a computer and LCD projector in my classroom.
- Students download a number of calculator programs from my calculator, including programs for Riemann Sums, Area between two curves, Euler's Method, and Slope Fields. These programs are designed to help students visualize the various concepts and to get a deeper understanding of calculus.
- Students are instructed throughout the course of the Four Functionalities allowed on the AP Exam with the graphing calculator including:
  - *Plot the graph of a function within an arbitrary viewing window.*
  - *Find the zeros of functions (solve equations numerically).*
  - *Numerically calculate the derivative of a function.*
  - *Numerically calculate the value of a definite integral.*
- I instruct students on the various software packages to illustrate volumes of solids, slope fields, and accumulation.
- During the course, problems will be represented and solved in four distinct ways: analytically, numerically, graphically, and verbally. Students will use a graphing calculator to determine the value of various limits, to determine the value of a derivative at a point, to find the value of a definite integral, to graph a function in various windows, and to solve a variety of equations, as well as explore concepts such as the limit of a function at a point.

CR1
The syllabus must list the title, author, and publication date of a college-level calculus textbook.

Textbooks

Primary Textbooks (1)

Larson, Hostetler, Edwards. Calculus of a Single Variable. Houghton Mifflin Company, 2006, 8th ed. ISBN 0618503048 **CR1**

Secondary Textbook

Stewart, James. Calculus. Brooks/Cole Publishing Company, 1999 ISBN: 0534359493

Resources and Supplementary Materials

College Board Special Focus Booklets including:

Differential Equations

Approximation

Infinite Series

The Fundamental Theorem on Calculus

College Board Curriculum Modules including:

Vectors

Volumes of Solids of Revolution

Extrema

Motion

Functions Defined by Integrals

Fundamental Theorem of Calculus

Reasoning from Tabular Data

Best, George. *Concepts and Calculators in Calculus.* Venture Publishing

Best, George. *AP Calculus and the TI-83 Graphing Calculator.* Venture Publishing.

Best, George. *Preparing for the AP Calculus Examination.* Venture Publishing

Bock, David. *Preparing for the AP Exam.* Barron's Educational Series.

Crawford, Debra. *Work Smarter Not Harder.* Venture Publishing.

Lederman, David. *Multiple-Choice and Free-Response Questions in Preparation for the AP Calculus AB Examination.* D&S Marketing Systems.

AP Calculus AB and BC Course and Exam Description (CED)

Teaching AP Calculus, D&S Marketing Systems, Inc., Lin McMullin, 2nd ed.

Software:

Best, George. *Best Grapher.*

Bradford, William. *Calculus AB Test Bank.*

Desmos

Weeks, Audrey. *Calculus in Motion.*

- Previously Published AP Multiple-Choice and Free-Response Questions including the 1997, 1998, 2003, 2008, 2012 released AP Exams
- AP Professional Development Workshops and Institute materials
- AP Central® website and AP Calculus OTC
- TI-83+ and TI-84 graphing calculators

Assessment

Students are assessed using several methods. The math department counts daily homework as 10% of a student's grade. The other 90% is a combination of quizzes, labs, projects, and unit tests. I will use the Personal Progress Checks (PPCs) designed by the College Board as formative assessments during the course of the 10 units to help students and me better understand what concepts my students are struggling with. The unit tests contain a no calculator section and a calculator section consistent with the AP Calculus BC Exam. Weekly labs consist of graphical, numerical, and analytical components and a written conclusion. Free-response questions are graded similar to the AP Exam. A midyear exam is given at the end of the first semester. Just before the AP Exam in May, students are given an entire AP BC Calculus practice exam, which is graded like the actual exam using the scoring guidelines published by the College Board. This is counted as their final exam grade for the year.

Because the mathematical communication component is so important in this class, students are strongly encouraged to do test corrections for every exam. These test corrections are an integral component of the learning process for this AP course and will help students understand the required concepts, as well as how to effectively communicate their answers.

Post AP Exam

After the AP Exam, topics covered vary each year depending on the time remaining in the school year and the number of students in the class. One project requires two-student groups to present an appropriate lab demonstration to a math class of underclassmen. Other years we continue on with more calculus topics, including Volume by the Shell Method and other concepts not covered under the BC curriculum.

Unit 1: Limits and Continuity	
Topic	**Skill**
1.1 Introducing Calculus: Can Change Occur at an Instant?	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.2 Defining Limits and Using Limit Notation	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.3 Estimating Limit Values from Graphs	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.4 Estimating Limit Values from Tables	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.5 Determining Limits Using Algebraic Properties of Limits	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
1.6 Determining Limits Using Algebraic Manipulation	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
Complete Personal Progress Check MCQ Part A for Unit 1	
1.7 Selecting Procedures for Determining Limits	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
1.8 Determining Limits Using the Squeeze Theorem	3.C. Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
1.9 Connecting Multiple Representations of Limits	2.C. Identify a re-expression of mathematical information presented in a given representation.
1.10 Exploring Types of Discontinuities	3.B. Identify an appropriate mathematical definition, theorem, or test to apply.
1.11 Defining Continuity at a Point	3.C. Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
Complete Personal Progress Check MCQ Part B for Unit 1	
1.12 Confirming Continuity over an Interval	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
1.13 Removing Discontinuities	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
1.14 Connecting Infinite Limits and Vertical Asymptotes	3.D. Apply an appropriate mathematical definition, theorem, or test.

Unit 1: Limits and Continuity	
Topic	**Skill**
1.15 Connecting Limits at Infinity and Horizontal Asymptotes	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Check FRQ A for Unit 1	
1.16 Working with the Intermediate Value Theorem (IVT)	3.E. Provide reasons or rationales for solutions and conclusions.
Complete Personal Progress Checks MCQ C and FRQ B for Unit 1	
Take Unit 1 Test	

Unit 2: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
2.1 Defining Average and Instantaneous Rates of Change at a Point	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
2.2 Defining the Derivative of a Function and Using Derivative Notation	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems. 4.C. Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
2.3 Estimating Derivatives of a Function at a Point	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist	3.E. Provide reasons or rationales for solutions and conclusions.
2.5 Applying the Power Rule	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ A for Unit 2	
2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
2.7 Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
2.8 The Product Rule	1.E. Apply appropriate mathematical rules or procedures, with and without technology.

Unit 2: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
2.9 The Quotient Rule	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ A for Unit 2	
2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
Complete Personal Progress Checks MCQ B and FRQ B for Unit 2	
Take Unit 2 Test	

Unit 3: Differentiation: Composite, Implicit, and Inverse Functions	
Topic	**Skill**
3.1 The Chain Rule	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g. Use the chain rule to find the derivative of a composite function).
3.2 Implicit Differentiation	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
3.3 Differentiating Inverse Functions	3.G. Confirm that solutions are accurate and appropriate.
3.4 Differentiating Inverse Trigonometric Functions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ B for Unit 3	
3.5 Selecting Procedures for Calculating Derivatives	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g. Use the chain rule to find the derivative of a composite function).
3.6 Calculating Higher Order Derivatives	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Checks MCQ and FRQ A for Unit 3	
Take Unit 3 Test	

Unit 4: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
4.1 Interpreting the Meaning of the Derivative in Context	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
4.3 Rates of Change in Applied Contexts Other Than Motion	2.A. Identify common underlying structures in problems involving different contextual situations.
4.4 Introduction to Related Rates	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
4.5 Solving Related Rates Problems	3.F. Explain the meaning of mathematical solutions in context.
4.6 Approximating Values of a Function Using Local Linearity and Linearization	1.F. Explain how an approximated value relates to the actual value.
Complete Personal Progress Check FRQ A for Unit 4	
4.7 Using L'Hospital's Rule for Determining Limits and Indeterminate Forms	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Checks MCQ and FRQ B for Unit 4	
Take Unit 4 Test	

Unit 5: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
5.1 Using the Mean Value Theorem	3.E. Provide reasons or rationales for solutions and conclusions.
5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points	3.E. Provide reasons or rationales for solutions and conclusions.
5.3 Determining Intervals on Which a Function is Increasing or Decreasing	2.E. Describe the relationships among different representations of functions and their derivatives.
5.4 Using the First Derivative Test to Determine Relative (Local) Extrema	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ A for Unit 5	
5.5 Using the Candidates Test to Determine Absolute (Global) Extrema	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
5.6 Determining Concavity of Functions Over Their Domains	2.E. Describe the relationships among different representations of functions and their derivatives.
Complete Personal Progress Check FRQ A for Unit 5	

Unit 5: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
5.7 Using the Second Derivative Test to Determine Extrema	3.D. Apply an appropriate mathematical definition, theorem, or test.
5.8 Sketching Graphs of Functions and Their Derivatives	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Check MCQ B for Unit 5	
5.9 Connecting a Function, Its First Derivative, and Its Second Derivative	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
5.10 Introduction to Optimization Problems	2.A. Identify common underlying structures in problems involving different contextual situations.
5.11 Solving Optimization Problems	3.F. Explain the meaning of mathematical solutions in context.
5.12 Exploring Behaviors of Implicit Relations	1.E. Apply appropriate mathematical rules or procedures, with and without technology. 3.E. Provide reasons or rationales for solutions and conclusions.
Complete Personal Progress Checks MCQ C and FRQ B for Unit 5	
Take Unit 5 Test	

Unit 6: Integration and Accumulation of Change	
Topic	**Skill**
6.1 Exploring Accumulations of Change	4.B. Use appropriate units of measure.
6.2 Approximating Areas with Riemann Sums	1.F. Explain how an approximated value relates to the actual value.
6.3 Riemann Sums, Summation Notation, and Definite Integral Notation	2.C. Identify a re-expression of mathematical information presented in a given representation.
6.4 The Fundamental Theorem of Calculus and Accumulation Functions	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
6.5 Interpreting the Behavior of Accumulation Functions Involving Area	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Checks MCQ A for Unit 6	
6.6 Applying Properties of Definite Integrals	3.D. Apply an appropriate mathematical definition, theorem, or test.

Unit 6: Integration and Accumulation of Change	
Topic	**Skill**
6.7 The Fundamental Theorem of Calculus and Definite Integrals	3.D. Apply an appropriate mathematical definition, theorem, or test.
6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation	4.C. Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
6.9 Integrating Using Substitution	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ A for Unit 6	
6.10 Integrating Functions Using Long Divisions and Completing the Square	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ B for Unit 6	
6.11 Integrating Using Integration by Parts	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
6.12 Using Linear Partial Fractions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
6.13 Evaluating Improper Integrals	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
6.14 Selecting Techniques for Antidifferentiation	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g. Use the chain rule to find the derivative of a composite function).
Complete Personal Progress Checks MCQ C and FRQ B for Unit 6	
Take Unit 6 Test	

Unit 7: Differential Equations	
Topic	**Skill**
7.1 Modeling Situations with Differential Equations	2.C. Identify a re-expression of mathematical information presented in a given representation.
7.2 Verifying Solutions for Differential Equations	3.G. Confirm that solutions are accurate and appropriate.
7.3 Sketching Slope Fields	2.C. Identify a re-expression of mathematical information presented in a given representation.
7.4 Reasoning Using Slope Fields	4.D. Use appropriate graphing techniques
7.5 Approximating Solutions Using Euler's Method	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ A for Unit 7	

Unit 7: Differential Equations	
Topic	**Skill**
7.6 Finding General Solutions Using Separation of Variables	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQs A and B for Unit 7	
7.8 Exponential Models with Differential Equations	3.G. Confirm that solutions are accurate and appropriate.
7.9 Logistic Models with Differential Equations	3.F. Explain the meaning of mathematical solutions in context.
Complete Personal Progress Check MCQ B and FRQ B for Unit 7	
Take Unit 7 Test	

Unit 8: Applications of Integration	
Topic	**Skill**
8.1 Finding the Average Value of a Function on an Interval	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
8.2 Connecting Position, Velocity, and Acceleration Using Integrals	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts	3.D. Apply an appropriate mathematical definition, theorem, or test.
8.4 Finding the Area Between Curves Expressed as Functions of x	4.C. Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
8.5 Finding the Area Between Curves Expressed as Functions of y	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ A for Unit 8	
8.6 Finding the Area Between Curves That Intersect at More Than Two Points	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
8.7 Volumes with Cross-Sections: Squares and Rectangles	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ A for Unit 8	
8.8 Volumes with Cross-Sections: Triangles and Semicircles	3.D. Apply an appropriate mathematical definition, theorem, or test.
8.9 Volume with Disc Method: Revolving Around the x – or y – axis	3.D. Apply an appropriate mathematical definition, theorem, or test.

Unit 8: Applications of Integration	
Topic	**Skill**
8.10 Volume with Disc Method: Revolving Around Other Axes	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
8.11 Volume with Washer Method: Revolving Around the x – or y – axis	4.E. Apply appropriate rounding procedures.
8.12 Volume with Washer Method: Revolving Around Other Axes	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
8.13 The Arc Length of a Smooth, Planar Curve and Distance Traveled	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ B and FRQ B for Unit 8	
Take Unit 8 Test	

Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions	
Topic	**Skill**
9.1 Defining and Differentiating Parametric Equations	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
9.2 Second Derivatives of Parametric Equations	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
9.3 Finding Arc Lengths of Curves Given by Parametric Equations	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
9.4 Defining and Differentiating Vector-Valued Functions	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
9.5 Integrating Vector-Valued Functions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ A for Unit 9	
9.6 Solving Motion Problems Using Parametric and Vector-Valued Functions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
9.7 Defining Polar Coordinates and Differentiating in Polar Form	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Checks FRQ A and FRQ B for Unit 9	

Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions	
Topic	**Skill**
9.8 Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve	3.D. Apply an appropriate mathematical definition, theorem, or test.
9.9 Find the Area of the Region Bounded by Two Polar Curves	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ B for Unit 9	
Take Unit 9 Test	

Unit 10: Infinite Sequences and Series	
Topic	**Skill**
10.1 Defining Convergent and Divergent Infinite Series	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.2 Working with Geometric Series	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.3 The nth Term Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.4 Integral Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.5 Harmonic Series and p – Series	3.B. Identify an appropriate mathematical definition, theorem, or test to apply.
Complete Personal Progress Check MCQ A for Unit 10	
10.6 Comparison Tests for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.7 Alternating Series Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.8 Ratio Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.9 Determining Absolute or Conditional Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.10 Alternating Series Error Bound	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ B for Unit 10	
10.11 Finding Taylor Polynomial Approximations of Functions	3.D. Apply an appropriate mathematical definition, theorem, or test. 2.C. Identify a re-expression of mathematical information presented in a given representation.
10.12 Lagrange Error Bound	1.F. Explain how an approximated value relates to the actual value.
10.13 Radius and Interval of Convergence of Power Series	2.C. Identify a re-expression of mathematical information presented in a given representation.

Unit 10: Infinite Sequences and Series	
Topic	**Skill**
10.14 Finding Taylor or Maclaurin Series for a Function	2.C. Identify a re-expression of mathematical information presented in a given representation.
Complete Personal Progress Check FRQ A for Unit 10	
10.15 Representing Functions as Power Series	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ C and FRQ B for Unit 10	
Take Unit 10 Test	

Review for AP Exam (2–3 weeks)

- Complete review of the proceeding major topics/concepts.
- Students work through problems by using published review books and College Board released MC, FRQs, and practice exams.
- Nightly practice problems from released AP Exams, including both multiple- choice and free-response questions.
- A complete practice exam (most recently released public exam) given over several class periods. This exam is graded just like the AP Exam using released scoring rubrics.

Teaching Strategies

One of the major outcomes of this course is for students to be able to work with functions represented in a variety of ways: graphically, numerically, analytically, and verbally. This is accomplished in a variety of methods, including daily homework presentations by students, weekly labs, take-home problem sets with written justifications using correct mathematical nomenclature, and group projects.

All exams are modeled after the AP Exam, including multiple-choice and free-response questions. All exams have both a calculator and a no calculator active section. Students are taught to round once during a problem, at the end of the problem and to three decimal places. Students are taught to develop connections/relationships between the three major themes of the course: change/limits/analysis of functions. These three themes are woven throughout the course as described in the CED. Students use their calculators to explore concepts and solidify their learning experiences. Students are exposed to a multitude of problems from various textbooks, review books, and other sources.

Activities/Projects

Students work on a number of activities/projects throughout the course to reinforce concepts and the Rule of Four. The activities below address the Four Mathematical Practices:

Mathematical Practice #1: Implementing Mathematical Processes

This activity is designed for the Chain Rule to help students develop their own understanding of this concept. I write $y = \sin(x)$ on the board and I ask them for the derivative, which at this point in the course should be easy for them. I then write a 1 after the answer $y' = \cos(x) * 1$.

Many students ask: Why did I write a 1 at the end of the answer? We then talk about this and then I ask them to take the derivative of $y = \sin(3x)$ and see what I get from the students. If they can see the pattern, and they usually do, we talk about this as being a very simple composition of functions and then move on to a more difficult situation, namely the derivative of $f(g(x))$.

After we have found the general formula for the Chain Rule, I give the students a worksheet with several functions expressed analytically. They must decide which functions require the Chai n Rule, which require the Product Rule, and which require neither rule and then manually compute the derivative of each function. **(1.C, 1.E)** **CR3**

Mathematical Practice #2: Connecting Representations

This is an easier classroom activity and works best for larger classes. I have eight small pieces of paper for each of five functions (for a total of 40 pieces of paper): the graph of the function and the graph of its first derivative, a tabular representation of the function and of its derivative, an analytical expression of the function and of its derivative, and a verbal description of the function and of its first derivative. Students try to match the 40 pieces of paper into the correct five sets of eight pieces each. I then have the students divide into pairs; each student must explain to their partner the connection between at least two different representations in each set (such as the connection between the graph of the function and the verbal description of its derivative). **(2.B, 2.C, 2.E)** **CR4**

Mathematical Practice #3: Justification

This activity is designed for students to apply the Mean Value Theorem in real- world situations. After introducing the MVT and discussing the conditions that need to be met to apply this theorem (continuous on [a,b] and differentiable on (a,b)), students are asked to come up with one real-world situation that satisfies the hypotheses of the MVT and one that does not satisfy the hypotheses. Students then explain orally to the class what their answers represent in context and explain their reasoning for whether the hypotheses are met or not. **(3.C, 3.E)** **CR5**

Mathematical Practice #4: Communication and Notation

In this activity, the students practice notational fluency and appropriate mathematical language. Each student writes a verbal description of a rate of change (such as "The rate of change of the temperature of the pie is proportional to the difference between the temperature of the pie and the temperature of the room") and gives it to their partner, who must translate the description into a differential equation using proper notation. For that night's homework assignment, each student must submit a written explanation of how language in the verbal description suggested their differential equation. **CR6** For the next night's homework assignment, they are asked to use calculus to solve the differential equation given an initial condition. The work and notation are checked. **(4.A, 4.C)**

Additional Activities

1. The students are given two complicated functions expressed analytically that represent the rate of change of the populations of wolves and coyotes. They must first use the calculator to draw the graphs, then use the calculator's equation solver to find where the graphs intersect, and third use the calculator's numerical differentiation feature to estimate the slope of each graph at the intersection point. Lastly, they must set-up definite integrals to find the net increase or decrease in each population over a given time period and solve with the calculator's numerical integration feature. **CR7**
2. The students are asked to graph various functions and their derivatives on the calculator in order to explore the relationship between the graph of the function and the graph of its derivative and discover any useful connections (such as that the graph of f' is increasing where the graph of f is concave up). **CR7**

CR3
The syllabus must include a description of at least one activity in which students use two or more skills under Mathematical Practice 1. The activity or activities must be labeled with the corresponding skill(s).

AND

One of those activities must incorporate the portion of Skill 1.E in which students apply appropriate mathematical rules or procedures without technology.

CR4
The syllabus must include a description of at least one activity in which students work with multiple representations. Each of the four representations (analytical, numerical, graphical, and verbal) must be in at least one of the provided activities.

AND

There must be evidence of a connection between at least two different representations in at least one of the provided activities aligned with Skills 2.C, 2.D, or 2.E.

The activity or activities must be labeled with the corresponding skill(s).

3. Students are introduced to using power series to estimate rational and trigonometric functions using calculator-graphing skills. Students are initially given the Taylor Series for a specific function. The power series is graphed term by term on the same viewing rectangle as the specified functions. Students visually see the power series approximation becomes a better estimate of the function as more terms are added.
4. An assignment has two polar curves. Students are asked to find the intersection points and the area closed by the two curves, $\frac{dr}{dQ}$ or $\frac{dy}{dx}$, and interpret the meaning in the context of the problem. Students are also asked to use their graphing calculator to perform the numerical integration of $\frac{dr}{dQ}$ or $\frac{dy}{dx}$.
5. Students will work independently and then compare their answers to identify when integration by parts is an appropriate strategy for integrating an expression and what they will choose for u and dv. They will then use integration by parts to find the integrals. We will also have a discussion covering what to do when the choices generate integrands such that we again would need to use integration by parts. Their homework will be a worksheet requiring them to select and apply the appropriate integration technique, including substitution and integration by parts. **CR3**
6. In this calculator-active, small-group activity, students will become familiar with the graphing of trigonometric and rational functions in a real-world context. Student will find the area bound by trigonometric curves and volume of curves using cross sections to the x-axis. Students will integrate both trigonometric functions and rational functions in order to find zeros for displacement, maximums, minimums, and average values. These values will be presented with explanations to address specific questions concerning the cooling of a house, total cost of cooling a house over a particular day, number of people entering a park, dollars collected for admission to a park, and predicting when the number of people in the park is a maximum. **CR8**

CR5

The syllabus must include a description of at least one activity in which students use two or more skills under Mathematical Practice 3. The activity or activities must be labeled with the corresponding skill(s).

AND

One of those skills must be 3.C.

AND

One of those skills must be either 3.E, or 3.F.

CR6

The syllabus must include a description at least one activity in which students are given the opportunity to communicate their understanding of calculus concepts, processes, or procedures using appropriate mathematical language. (Skill 4.A)

AND

The syllabus must include a description of at least one activity in which students demonstrate notational fluency by either connecting different notations for the same concept or using appropriate mathematical notation in applying procedures. (Skill 4.C)

The activity or activities must be labeled with the corresponding skill(s).

CR8

The syllabus must provide a description of at least one activity requiring students to apply their knowledge of AP Calculus concepts to solve real-world problems.